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TEACH

MODULE BIOMECHANICS: FOUNDATIONS OF BIOMECHANICS APPLIED TO THE LOCOMOTOR SYSTEM

Didactic Unit A: MOVEMENTS

Politechnika Śląska

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AMSE THE ASSOCIATION OF MEDICAL SCHOOLS IN EUROPE

Index

1. Objectives

- To know the foundations of movements (kinematics) and differences with the causes that produce them (kinetics).
- To describe the fundamental concepts in linear movements: position, displacement, velocity and acceleration.
- To define fundamental concepts in circular movements: angle, angular velocity and angular acceleration.
- To learn foundations of human motion: movement planes and Euler angles.

2. Foundations of movements: Kinematics and kinetics

The study of the movement of living things using the science of mechanics is performed through biomechanics.

2.1. Kinematics

The part of the mechanics that analyse the motion of bodies (going in straight line or turning) without considering the forces that cause them is called **kinematics.** Kinematics describes the motion of a body through different variables:

- Its positions and trajectory (where is the body located in every moment?)
- Its velocity (How fast does it move?)
- Its acceleration (How does the velocity change?)

Examples of kinematic variables related to movement are position, displacement/trajectory, time, angle/range of motion, velocity and acceleration.

In summary:

Kinematics answer the questions about how a body moves.

2.2. Kinetics

The part of mechanics that study the causes of bodies motion (forces) is called **kinetics*.** Kinetics describes the forces that act over a body to produce movement.

Examples of kinetic variables related to movement are any type of force (friction, ground reaction, gravitational, etc.), work, momentum, torque, energy, power and resistance.

In summary:

Kinetics answer the questions about why a body moves.

In most biomechanical studies, the body being analysed is assumed to be rigid, so deformations in its shape can be ignored. Therefore, the skeletal system is considered as a rigid-body whose motion is based on rigid-body mechanical principles. This assumption solves important and difficult mathematical and modelling calculations without loss of exactitude and accuracy [1].

**Kinetics and Dynamics are often used interchangeably.*

As part of your theoretical training it is recommend watching a video about kinematic and kinetic differences. You can access to some example videos through the following links: https://ocw.mit.edu/courses/physics/8-01sc-classical-mechanics-fall-2016/week-1-kinematics/week-1-introduction/

The material that the hyperlinks lead to, is public and available for viewing online. It has been selected for its adequacy with the subject covered in this teaching unit (movements), after performing a search using the terms "Classical Mechanics", on the web indicated above. Like these, you can find and revise other interesting public didactic videos by using the same searching terms.

2.3. Basic Concepts

2.3.1 Scalar Vs Vector

Mechanical variables (kinematic and kinetic) previously named can be defined only by their magnitudes or it could also require a direction.

Variables that can be represented by a number (and its measurement unit) are called **scalar**.

Examples of scalar variables and their units related to movement are mass (Kg), time (s), range of motion/angle (º), power (W), energy (J) and temperature (ºC).

Variables that are represented by both, their magnitudes and a direction associated with them, are called **vectors.**

Examples of vector variables related to movement are position, displacement, velocity, acceleration, force and torque/momentum.

Graphically, vectors are represented by an arrow.

2.3.4.1. Vector properties

The length of the arrow that represents a vector is proportional to the numerical magnitude of that vector (denoted by $|\vec{V}|$) and the arrowhead indicates the direction and sense of the vector (Figure 1).

Example:

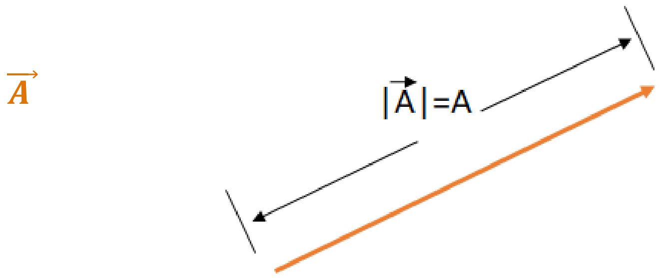


Figure 1: Representation of vector $\vec{A}$

Vectors complies with the traditional mathematical rules about addition and scalar multiplication (Table 1 and Table 2) [3].

Table 1: Vector Addition properties

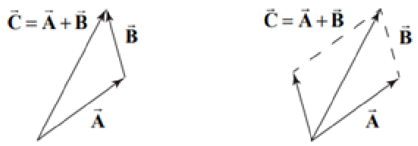
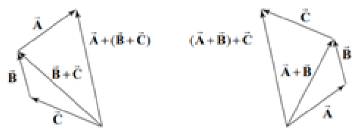
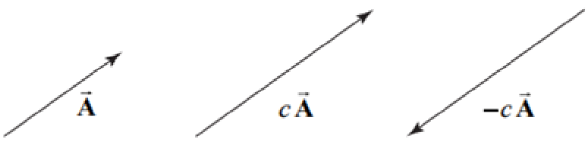
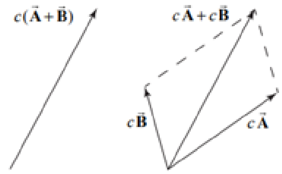
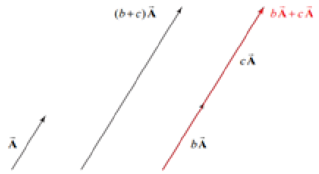
Vector Addition			
			
Vector Commutativity		Vector Associativity	
Identity Element for Vector Addition	$\vec{A}+\vec{0}=\vec{0}+\vec{A}=\vec{A}$	Inverse Element for Vector Addition	$\vec{A}+(-\vec{A})=\vec{0}$

Table 2: Scalar Multiplication of vector properties

Scalar Multiplication of Vectors			
			
Associative Law for Scalar Multiplication	$b(c\vec{A}) = (bc)\vec{A} = (cb\vec{A}) = c(b\vec{A})$	Distributive Law for Vector Addition	
Distributive Law for Scalar Multiplication		Identity Element for Scalar Multiplication	$\vec{A}+(-\vec{A})=\vec{0}$

2.3.2. Coordinate system

In any biomechanical analysis, a reference frame is required. It allows the description of the one-two or three-dimensional position of the body under analysis. The typical reference frame used in biomechanics is called **Cartesian coordinate system** (Figure 2). It consists of a set of mutually perpendicular axes, which meet at a common point, its origin 0.

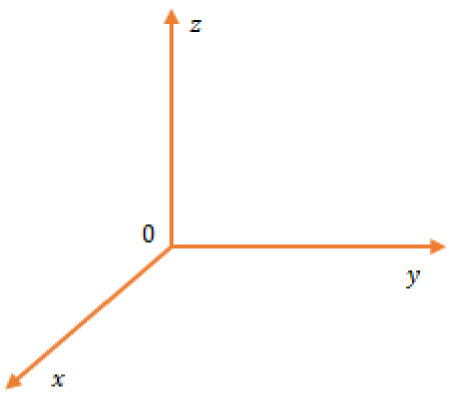
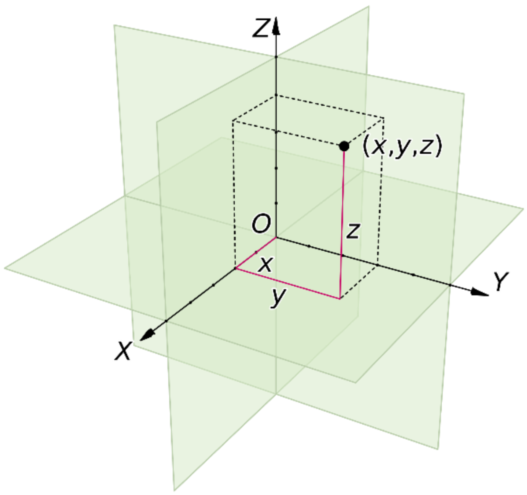


Figure 2: Cartesian Coordinate system

Movements in our world are performed in 3D, therefore this coordinate system has three axes named as follows: x-axis, y-axis and z-axis. There is not a worldwide standard that determines which axis corresponds to which letter (and its positive and negative direction) but to stablish a convention must be indeed determined.

The coordinates of a point are usually written as three numbers surrounded by parentheses and separated by commas (x,y,z). Every coordinate system has a set of unitary vectors associated ($\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$) that defines the direction of each vector component in x, y and z-axis. In Figure 3, point P whose coordinates in 3D space are 2,3,4 is represented. Planes formed by two axes can be also observed: XY-plane, YZ-plane and XZ-plane.



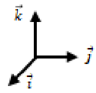


Figure 3: Point (2,3,4) represented by Cartesian Coordinate system. Extracted from: https://commons.wikimedia.org/wiki/File:Coord_system_CA_

Taking P= (2,3,4), let´s go to extract useful information:

Point P is a point in 3D space because all its components are different to 0.

Vector component in x-axis= 2 in positive direction.

Vector component in y-axis= 3 in positive direction.

Vector component in z-axis= 4 in positive direction.

Representation of P as a vector using unit vectors:

$$\vec{P} = 2\vec{\imath} + 3\vec{\jmath} + 4\vec{k}$$

The magnitude of $\vec{P}$ can be obtained calculating its module, that, by definition:

$$\left|\vec{P}\right| = \sqrt{P_x^2 + P_y^2 + P_z^2}$$

Being:

P_x= Component of P in x-axis.

P_y= Component of P in y-axis.

P_z= Component of P in z-axis.

So:

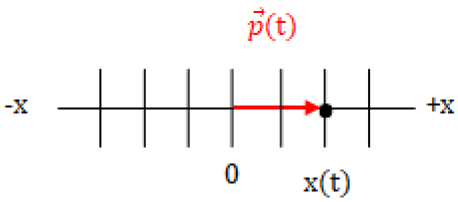
$$\left|\vec{P}\right|: \sqrt{2^2 + 3^2 + 4^2} = 5.38$$

3. Fundamental concepts in linear movements: position, displacement, velocity and acceleration

3.1 Fundamental concepts in linear movements in one dimension:

Motion of a body implies a change of position with respect to a reference frame (in this case, the Cartesian coordinate system). So, firstly the initial position of that body with respect to the origin 0 should be determined.

Consider a body moving in one dimension (in x-axis) as a function of time (t). Its initial position is defined by a **position coordinate** and denoted by $x(t)$. This coordinate can be positive or negative depending where the body is located. As it was seen in the previous example, position coordinate has a magnitude and direction, therefore is a vector, which is denoted as a **position vector** (Figure 4), named simply vector [4].



$$\vec{p}(t) = x(t)\vec{\imath} = 2\vec{\imath}$$

Figure 4: Representation of position vector $\vec{p}$ in Cartesian coordinate

The change of position of a body during a time interval [t_1, t_2] is called **displacement vector** (Figure 5). The displacement of a body as a function of a time interval is the vector difference between the two position vectors in t_1 and t_2 [4].

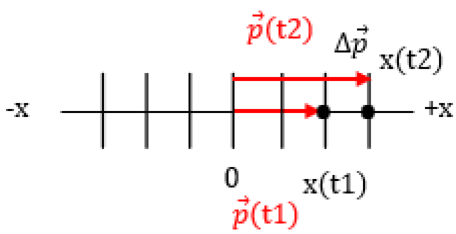


Figure 5: Representation of the displacement of a vector $\Delta\,\vec{p}(t)$ in Cartesian coordinate system

$$\Delta\,\vec{p}(t) = \vec{p}(t_2) - \vec{p}(t_1) = \big(x(t_2) - x(t_1)\big)\vec{\imath} = 3\vec{\imath} - 2\vec{\imath} = i$$

**In science field, symbol Δ means difference between two values or the rate of change.*

It is commonly known that velocity is the result of dividing distance over time, but the resulting number without defining a direction, gives only an idea about a magnitude (scalar) not being really useful in biomechanics.

In physics, velocity is the rate of change of displacement. This rate requires to know the direction of velocity (in which axis is acting), so it is indeed considered as a vector.

So, considering the example of Figure 5, velocity of $\vec{p}(t)$ in x-axis:

$$v_{\vec{p}} = \frac{\Delta\, p(t)}{\Delta(t)}\vec{\imath}$$

This equation defines the **average velocity** in a time interval. In order to know the **instantaneous velocity** at a certain time:

$$\vec{v}(t) = \frac{dp}{dt}$$

By definition, instantaneous velocity is a vector resulting of the derivative of the position vector with respect to time.

The international system (SI) units for velocity are meters per second $\left[\frac{m}{s}\right]$.

Acceleration is the rate of change of velocity. It means that in the case that the velocity does not change (it is constant), the acceleration is equal to zero. Considering example of figure 5, acceleration of $\vec{p}(t)$ in x-axis:

$$a_{\vec{p}} = \frac{\Delta\, v(t)}{\Delta(t)}\vec{\imath}$$

By mathematical definition, acceleration is a vector resulting of the derivative of the velocity vector or the second derivative of the position vector with respect to time.

$$\vec{a}(t) = \frac{dv}{dt} = \frac{d^2p}{dt^2}$$

The international system (SI) units for acceleration are meters per square seconds $\left[\frac{m}{s^2}\right]$.

In the case that acceleration is constant, these useful equations can be used:

$$p = vt$$
$$v = v_0 + at$$
$$p = v_0 t + \frac{1}{2}at^2$$
$$\Delta p = \frac{(v_0 + v)}{2}t$$

3.2 Fundamental concepts in linear movements in more than one dimension [5]:

In order to describe the motion of a body in two or three dimensions, each dimension is considered independently. Therefore, considering $\vec{p}(t)$ as a **position vector** in two dimensions (Figure 6):

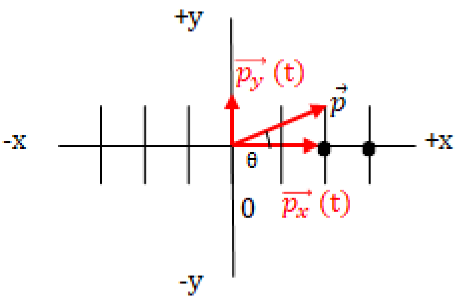


Figure 6: Representation of two dimensions position vector p in Cartesian coordinate system

$$\vec{p}(t) = p_x(t)\vec{\imath} + p_y(t)\vec{\jmath} = p\cos\theta\,\vec{\imath} + p\sin\theta\vec{\jmath}$$

The vector position should be decomposed in its x, y and z components. In this case, p_x is the p-component in x-axis and p_y is the p-component in y-axis.

The rest of equations seen previously in one dimension are rewritten to define the motion in more than one dimension:

$$v_{\vec{p}} = \frac{\Delta\, p_x(t)}{\Delta(t)}\vec{\imath} + \frac{\Delta\, p_y(t)}{\Delta(t)}\vec{\jmath} \qquad \vec{v}(t) = \frac{dp_x}{dt} + \frac{dp_y}{dt} = v_x(t)\vec{\imath} + v_y(t)\vec{\jmath}$$

$$a_{\vec{p}} = \frac{\Delta\, v_x(t)}{\Delta(t)}\vec{\imath} + \frac{\Delta\, v_y(t)}{\Delta(t)}\vec{\jmath} \qquad \vec{a}(t) = \frac{dv}{dt} = \frac{d^2p}{dt^2} = a_x(t)\vec{\imath} + a_y(t)\vec{\jmath}$$

Notice that a positive sign has been used in the equations because position, displacement, velocity and acceleration vectors represented in one and two dimensions are located in the positive x and y-axis. It would change to negative sign according to where they are located.

4. Fundamental concepts in circular movements: angle, angular velocity and angular acceleration.

Circular movements can be defined by a similar way that linear movements.

Consider a point P which is moving in a circular motion. In figure 7 the position of P at a certain moment is described. The vector position $\vec{r}(t)$ and the angle θ (angular displacement) define where point P is located with respect to the x and y axis.

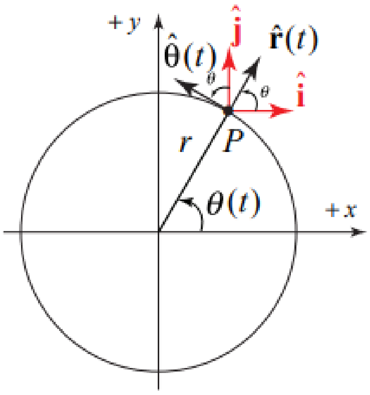


Figure 7: Position vector r(t) of a body moving in a circular motion of radius r. Extracted from: https://ocw.mit.edu/courses/physics/8-01sc-classical-mechanics-fall-2016/readings/MIT8_01F16_chapter6.2.pdf [6]

Therefore, the equation of the vector position with respect to the x-axis is defined by:

$$\vec{r}(t) = cos\theta(t)\vec{\imath} + sin\theta(t)\vec{\jmath}$$

Angular velocity (ω) is equivalent to linear velocity in linear movements. The angular velocity is the magnitude of the rate of change of angle θ with respect to time. So, in the case described in figure 7 [8]:

$$\omega = \frac{d\theta(t)}{dt}$$

Angular and linear velocity are related by the radius (r):

$$\omega = \frac{v}{r}$$

The international system (SI) units for angular velocity are meters per second $\left[\frac{rad*}{s}\right]$.

**rad is the acronym of radians*

Notice that angle and angular velocity are scalar magnitudes, they are not vectors.

Angular acceleration (α) is equivalent to linear acceleration in linear movements. The angular acceleration is the magnitude of the rate of change of angular velocity ω with respect to time. So, in the case described in figure 7 [8]:

$$\alpha = \frac{d\theta^2(t)}{dt^2} = \frac{d\omega}{dt}$$

Angular (tangential) and linear acceleration are related by the radius (r):

$$\alpha = \frac{a_t}{r}$$

The international system (SI) units for angular acceleration are meters per square seconds $\left[\frac{rad}{s^2}\right]$.

In the case that angular acceleration is constant, these useful equations can be used [8]:

$$\theta = \omega t$$
$$\omega = \omega_0 + \alpha t$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

5. Foundations of human motion: movement planes and Euler angles

Human motions are performed in three dimensions, so movement planes in 3D are required to describe them. These planes are conformed by the three space axis: Vertical, longuitudinal and lateral axis. To define these planes, the anatomical position of the body has been considered (Figure 8).

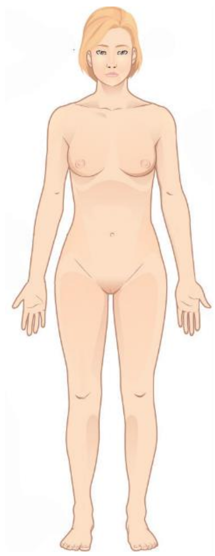


Figure 8: Anatomical position of the body. Extracted from: https://upload.wikimedia.org/wikipedia/commons/b/b2/Anatomical_position.jpg and modified by IBV.

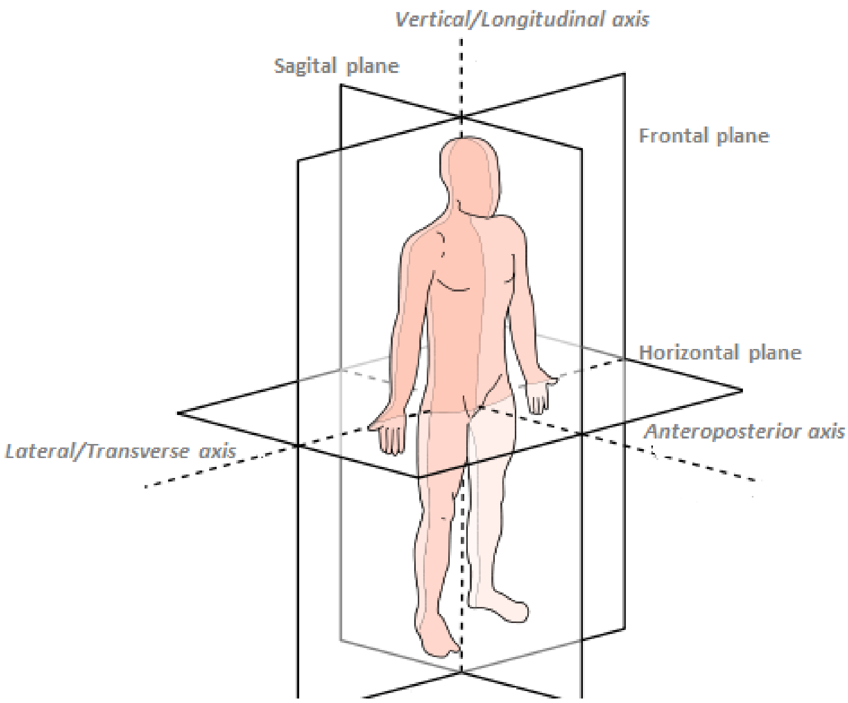


Figure 9: Axis and planes used in human motion. Extracted from https://es.wikipedia.org/wiki/Plano_anat%C3%B3mico and modified by IBV

In figure 10, the planes of movement are described. In each plane, the following movements are defined:

Horizontal plane: Rotation movements (Figure 10, left)

Frontal plane: Abduction and adduction movements (Figure 10,center)

Sagittal plane: Flexion and extension movements (Figure 10, right)

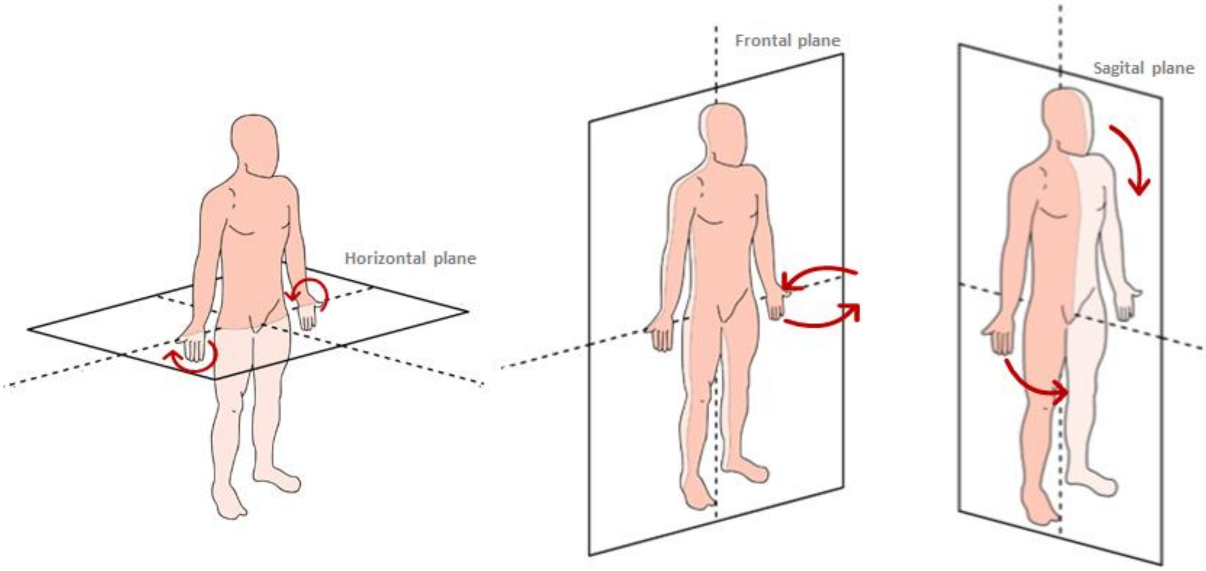


Figure 10: Planes and movements in each plane. Left: Rotation movements in horizontal plane; Centre: Abduction and adduction movements in frontal plane; Right: Flexion and extension movements in sagittal plane.

In order to describe the position of two bodies that are moving, **Euler´s method** is required. According to that method, any rotation might be described using three parameters that are called Euler angles. Let see an example (Figure 11)

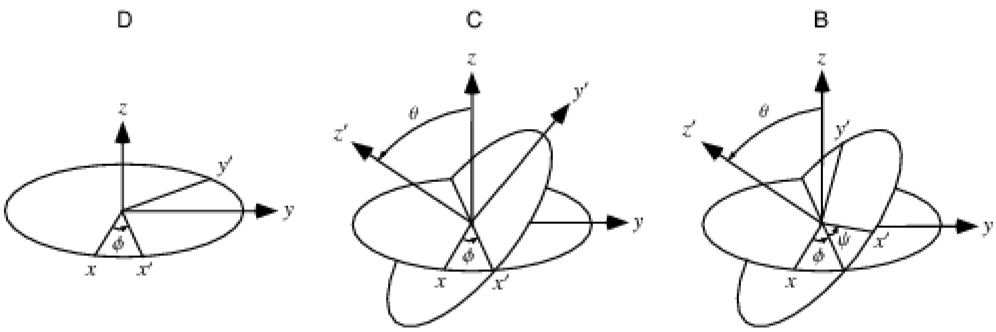


Figure 11: Euler angles. Extracted from: http://mathworld.wolfram.com/EulerAngles.html

The Euler's rotation angles are, by convention, defined by (Φ,θ,Ψ), where [7]:

- The first rotation is by an angle Φ about the z-axis using D (figure 11, left).

- The second rotation is by an angle θ in [0,π] about the former x-axis (now x') using C (Figure 11, center).

-The third rotation is by an angle Ψ about the former z-axis (now z') using B (Figure 11, right).

The difference with cartesian coordinate planes previously seen (Figure 9), is that the initial reference system used in Euler´s method can change in every rotation, so, it is closer to what happen in the real world when sometimes the reference system are not fixed.

The reason why Euler´s angles are explained is because they are very important in clinical evaluation. Movement´s planes have been previously defined considering anatomical position of the body. At the moment that a joint changes its position in the space, the movements defined in a plane could also change. It means that, for example, flexo-extension movements belong to sagittal plane considering anatomical position of the body, but it does not imply that flexo-extension movements are always performed in that plane (It will depend on the initial position of the joint).

6. References

The references should follow IEEE style. The order of the entries in the reference section should be in the order of appearance in the document [1].

[1] D.Knudson, fundamentals of Biomechanics. Cambrigde, 2007

[2] M. Nordin, V.H. Frankel, Basic biomechanics of the musculoskeletal system. Lippincott Williams & Wilkins, 2001.

[3] https://ocw.mit.edu/courses/physics/8-01sc-classical-mechanics-fall-2016/index.htm. Available in: https://ocw.mit.edu/courses/physics/8-01sc-classical-mechanics-fall-2016/readings/MIT8_01F16_chapter3.pdf

[4] https://ocw.mit.edu/courses/physics/8-01sc-classical-mechanics-fall-2016/index.htm. https://ocw.mit.edu/courses/physics/8-01sc-classical-mechanics-fall-2016/readings/MIT8_01F16_chapter4.2.pdf

[5] https://ocw.mit.edu/courses/physics/8-01sc-classical-mechanics-fall-2016/readings/MIT8_01F16_chapter5.1.pdf

[6] https://ocw.mit.edu/courses/physics/8-01sc-classical-mechanics-fall-2016/readings/MIT8_01F16_chapter6.2.pdf

[7] http://mathworld.wolfram.com/EulerAngles.html

[8] http://hyperphysics.phy-astr.gsu.edu/hbase/rotq.html